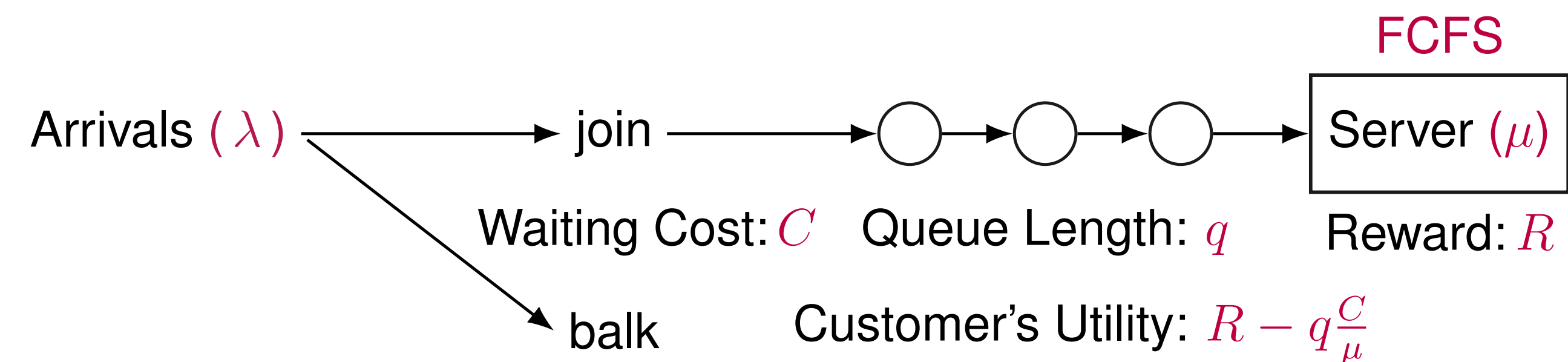




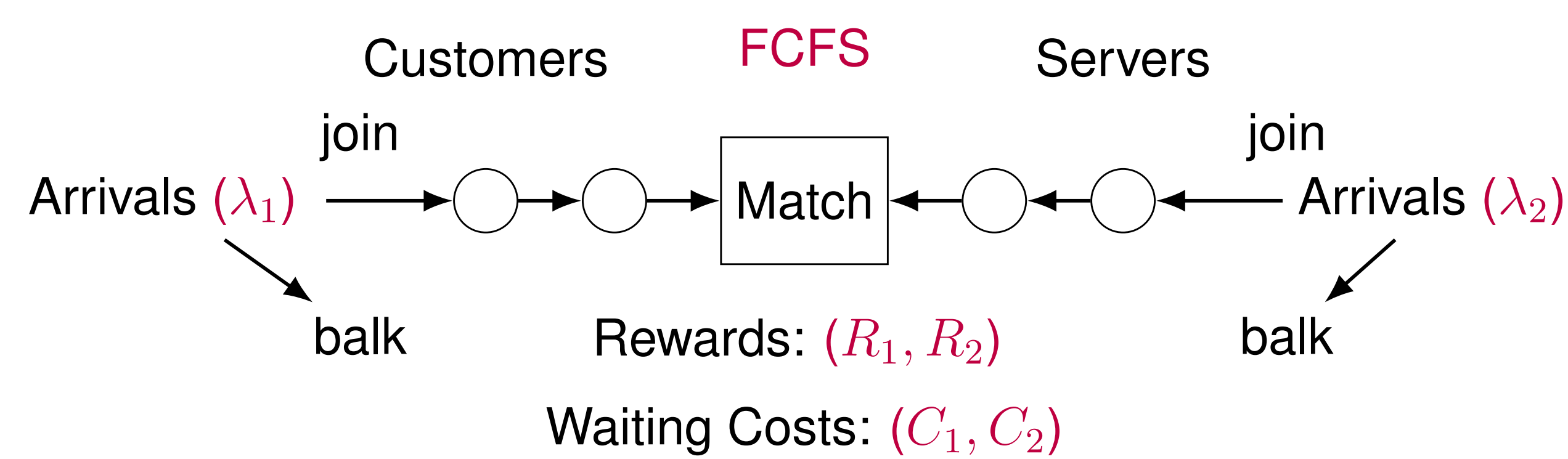
One-Sided Strategic Queueing (Naor 1969)



- **Equilibrium:** threshold strategy, join if $q < n^I = \frac{R\mu}{C}$
- **Welfare Maximization:** n^S
- Naor's Key Insight: **Over-Joining** ($n^S \leq n^I$) due to the **Negative Externality** on future customers
- **Pricing:**

$$p^S = R - n^S \frac{C}{\mu}$$

Model: Two-Sided Queues with Strategic Agents



- Zero matching time $\implies$ queue only on one side
- **Equilibrium:** Join if queue length $\leq n_i^I = \lfloor \frac{R_i \lambda_{-i}}{C_i} \rfloor$
- **Social Welfare:** jointly unimodal in thresholds, with a unique maximizer (n_1^S, n_2^S)

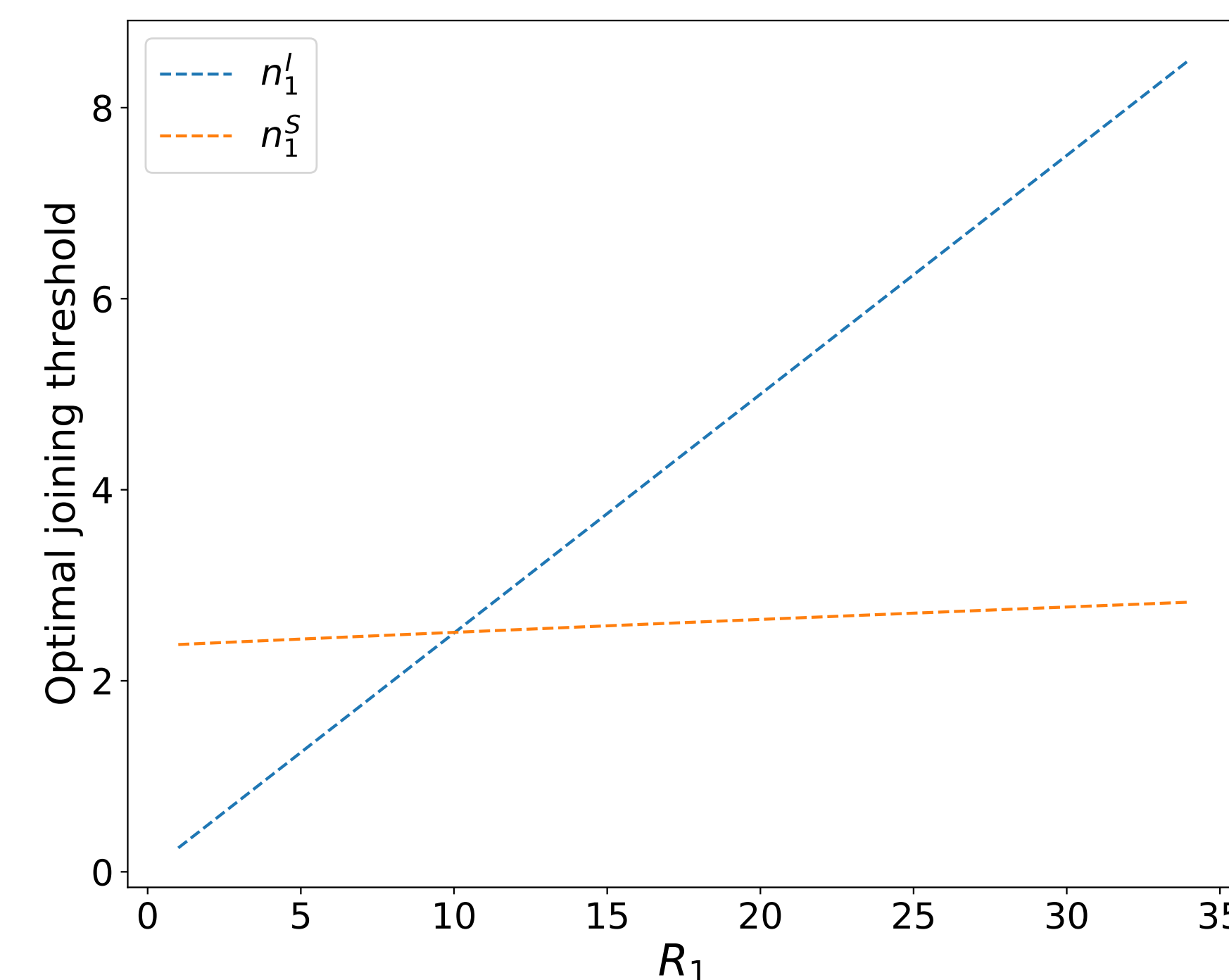
Research Questions

1. Do agents **over- or under-join** relative to welfare maximization?
2. Can **revenue-positive prices** induce welfare-maximizing thresholds?
3. **PoA:** How much efficiency loss arises from self-interested behavior?
4. What are the **profit-maximizing** thresholds?
5. **PoP:** How much efficiency loss arises from profit maximization?

Theorem 1 (Over/Under- Joining)

Under-joining possible due to **positive cross-side externality**.

Under-joining $\implies$ Socially optimal price negative: $p_i^S = \frac{C_i}{\lambda_{-i}}(n_i^I - n_i^S)$



Theorem 2 (Revenue-Positive Pricing)

The total price to induce n_1^S and n_2^S is nonnegative:

$$p_1^S + p_2^S \geq 0.$$

$$\text{Price of Anarchy (PoA)} = \frac{\text{Optimal Welfare}}{\text{Welfare Under Equilibrium}}$$

Theorem 3 (PoA Boundedness)

- $\lambda_1 \neq \lambda_2 \implies$ PoA is unbounded.
- $\lambda_1 = \lambda_2 \implies$ PoA is bounded.

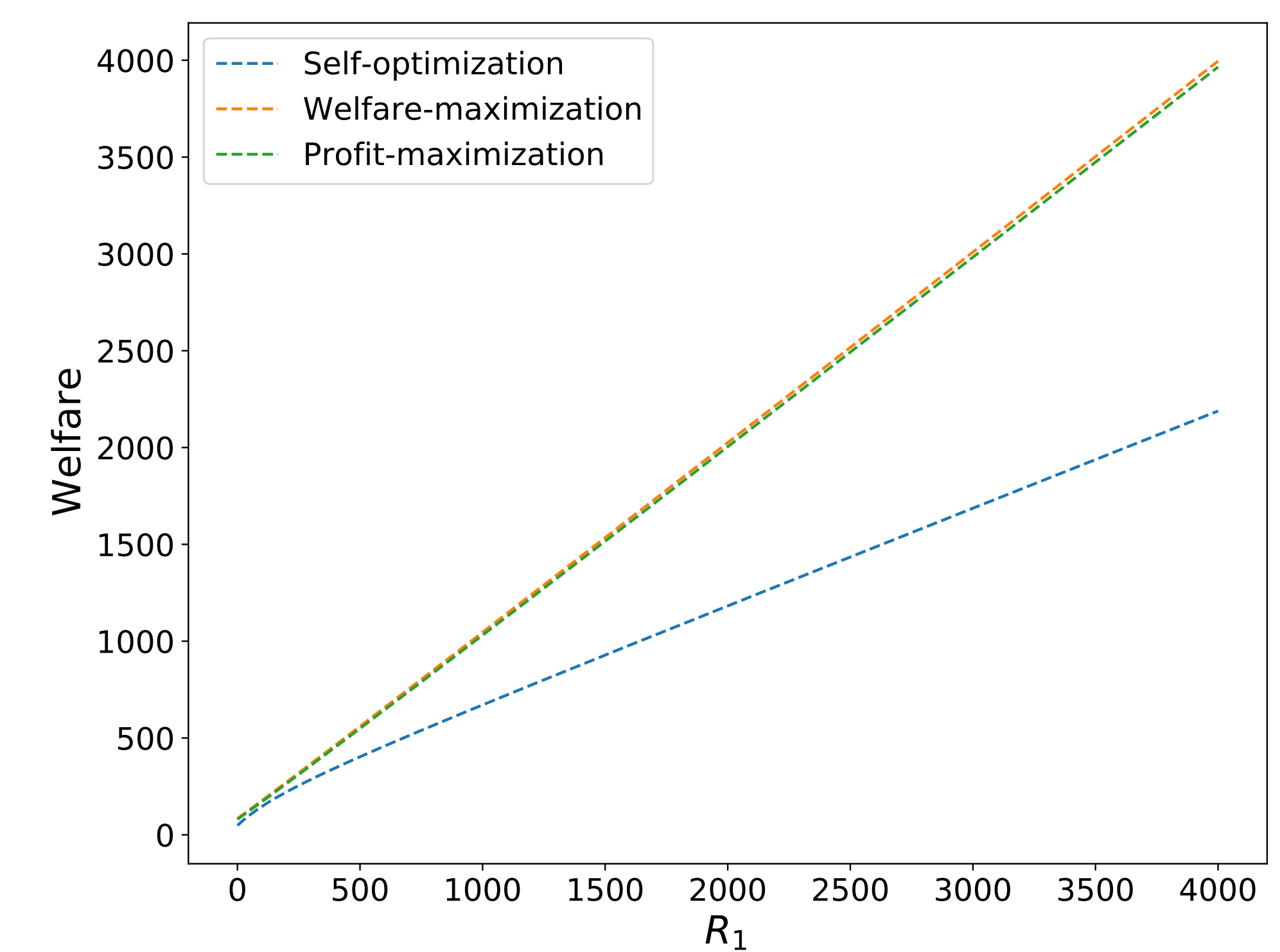
Theorem 4 (Profit Maximization Solution)

- System operates like a one-sided queue.
- The side with higher λC never waits.

$$\text{Price of Profit (PoP)} = \frac{\text{Optimal Welfare}}{\text{Welfare Under Profit Maximization}}$$

Theorem 5 (PoP Boundedness)

- PoP is always bounded.
- Under symmetric arrival rates: $\text{PoP} \rightarrow 1$ as $R_1 + R_2 \rightarrow \infty$.



Takeaways

Effect	One-sided	Two-sided
Negative externality	Yes	Yes
Positive externality	No	Yes
Over-joining likely	Yes	Not necessarily
Under-joining possible	No	Yes
Pigouvian toll	Always +	Possibly – (subsidy)
Sensitivity to asymmetry	No	Yes
Revenue positive pricing	Yes	Yes